Warm-up 4/19/17
Describe a pattern and give the next three terms.

1. 5, 10, 15, 20, 25, ... 30, 35, 40
add 5

2. 2, 4, 8, 16, 32, ... 64, 128, 256
mult. 2

9.4 Sequences

Sequence: An ordered sequence of numbers that can be finite or infinite.

1. $5, 10, 15, 20, 25$

2. $2, 4, 8, 16, 32, \ldots, 2^k, \ldots$

3. $\left\{\frac{1}{k}; k = 1, 2, 3, \ldots\right\}$

4. $\{a_1, a_2, a_3, \ldots, a_k, \ldots\} = \{a_k\}$

Find the first 6 terms and the 100th term of the sequence $\{a_k\}$ in which $a_k = k^2 - 1$

$a_1 = 1^2 - 1 = 0$
$a_2 = 2^2 - 1 = 3$
$a_3 = 3^2 - 1 = 8$
$a_4 = 4^2 - 1 = 15$
$a_5 = 24$
$a_6 = 35$
$a_{100} = 100^2 - 1$
$= 9{,}999$

Find the first 6 terms and the 100th term of the sequence defined recursively by the conditions:

$b_1 = 3$

$b_n = b_{n-1} + 2$ for all $n > 1$

3, 5, 7, 9, 11, 13

$2n+1$

$2(100)+1 = 201$

Limit of a Sequence

Let $\{a_n\}$ be a sequence of real numbers, and consider $\lim_{n\to\infty} a_n$. If the limit is a finite number L, the sequence converges and L is the limit of the sequence. If the sequence is infinite or nonexistent, the sequence diverges.

Determine whether the sequence converges or diverges.

1. $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{1}{n}, \ldots$ $\lim_{n\to\infty} \frac{1}{n} = 0$

$\lim(1) + \lim(\frac{1}{n})$

2. $\frac{2}{1}, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \ldots$ $\lim_{n\to\infty}\left(\frac{n+1}{n}\right) = \lim_{n\to\infty}\left(1 + \frac{1}{n}\right) = 1 + 0 = 1$

3. 2, 4, 6, 8, 10, ... diverges

4. -1, 1, -1, 1, ..., $(-1)^n$, ... diverges

Arithmetic Sequence

A sequence $\{a_n\}$ is an arithmetic sequence if it can be written in the form

$$\{a, a + d, a + 2d, \boxed{a + (n - 1)d}, \ldots\} \text{ for some constant } d.$$

explicit

The number d is called the common difference. Each term in an arithmetic sequence can be obtained recursively from its preceding term by adding d:

$$a_n = a_{n-1} + d \text{ (for all } n \geq 2)$$

Geometric Sequence

A sequence $\{a_n\}$ is a geometric sequence if it can be written in the form

$$\{a, a\cdot r, a\cdot r^2, \ldots, \boxed{a\cdot r^{n-1}}, \ldots\}$$ for some nonzero constant r.

explicit

The number r is called the common ratio. Each term in a geometric sequence can be obtained recursively from its preceding term by multiplying by r:

$$a_n = a_{n-1}\cdot r \text{ (for all } n \geq 2)$$

For each of the following sequences say whether it is arithmetic or geometric, then write an explicit and recursive rule for the nth term.

1. 3, 6, 12, 24, ...

geo.

$g_1 = 3$

$g_n = g_{n-1}\cdot 2$

$g_n = 3(2)^{n-1}$

2. -6, -2, 2, 6, 10, ...

arith.

$a_1 = -6$

$a_n = a_{n-1} + 4$

$a_n = -6 + 4(n-1)$

$= 4n - 10$

The second and fifth terms of a sequence are 3 and 24, respectively. Find explicit and recursive formulas for the sequence if it is

1. arithmetic

$a_1, 3, a_3, a_4, 24$

$+21$

$a_n = a_{n-1} + 7 \quad a_1 = -4$

$a_n = -4 + 7(n-1)$

$= 7n - 11$

2. geometric

$g_1, 3, g_3, g_4, 24$

2^3

$\cdot 8$

$g_n = g_{n-1} \cdot 2 \quad g_1 = 1.5$

$g_n = 1.5(2)^{n-1}$

Assignment: Pg. 676

#1-12e, 23-26, 39, 45