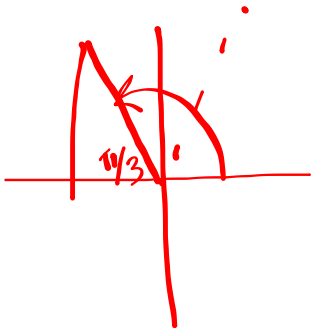
Warm-up 3/8/17

Use a sum or difference identity to find an exact value.

$\sin\frac{11\pi}{12}$



$\sin\left(\frac{2\pi}{3}+\frac{\pi}{4}\right)$

$\sin\frac{2\pi}{3}\cos\frac{\pi}{4}+\cos\frac{2\pi}{3}\sin\frac{\pi}{4}$

$\frac{1}{2}\cdot\frac{\sqrt{2}}{2}+\left(-\frac{\sqrt{3}}{2}\right)\cdot\frac{\sqrt{2}}{2}=\frac{\sqrt{2}-\sqrt{6}}{4}$

Formulas from Section 5-3

$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$

$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

5.4: Multiple-Angle Identities

$\sin 2u = \sin(u + u)$

$= \sin u \cos u + \cos u \sin u$

$= \sin u \cos u + \sin u \cos u$

$= 2\sin u \cos u$

$\cos^2 + \sin^2 = 1$

$\cos^2 = 1 - \sin^2$

so $\sin 2u = 2\sin u \cos u$

$\cos 2u = \cos(u + u)$

$= \cos u \cos u - \sin u \sin u$

$= \cos^2 u - \sin^2 u$

$1 - \sin^2 - \sin^2$

so $\cos 2u = \cos^2 u - \sin^2 u$

$= 1 - 2\sin^2 u$

$= 2\cos^2 u - 1$

If $\sin x = {}^3/_5$ and $\cos x = -{}^4/_5$, find the exact value of

$\sin 2x = 2(\sin x)(\cos x)$

$= 2\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right) = -\frac{24}{25}$

$\cos 2x = 1 - 2\sin^2 x$

$= 1 - 2\left(\frac{3}{5}\right)^2 = 1 - \frac{18}{25} = \frac{7}{25}$

Half-Angle Identities

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}} \qquad \cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$$

$$\tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u}$$

$1 - \frac{\sqrt{3}}{2}$

$\frac{2}{2} - \frac{\sqrt{3}}{2}$

Find the exact value of sin 15°, using the half-angle identity.

$\sin\left(\frac{30}{2}\right) = \pm\sqrt{\frac{1-\cos 30}{2}} = \sqrt{\frac{1-\sqrt{3}/2}{2}}$

$= \sqrt{\frac{2-\sqrt{3}}{2}\cdot\frac{1}{2}} = \frac{\sqrt{2-\sqrt{3}}}{2}$

Find the exact value of tan 22.5°.

$\tan\left(\frac{45}{2}\right) = \frac{\sin 45}{1+\cos 45} = \frac{\sqrt{2}/2}{1+\sqrt{2}/2} = \frac{\sqrt{2}/2}{\frac{2+\sqrt{2}}{2}}$

$\frac{\sqrt{2}}{2}\cdot\frac{2}{2+\sqrt{2}} = \frac{\sqrt{2}}{2+\sqrt{2}}$

Prove the identity. $\dfrac{\cot x - \tan x}{\cot x + \tan x} = \cos 2x$

$$\frac{\cos\cdot\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}\cdot\sin}{\cos\cdot\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}\cdot\sin} = \frac{\frac{\cos^2 x - \sin^2 x}{\cos x \sin x}}{\frac{\cos^2 x + \sin^2 x}{\cos x \sin x}}$$

$$\frac{\cos 2x}{\cos x \sin x}\cdot\frac{\cos x \sin x}{1} = \cos 2x$$

Prove the identity. $\csc 2x = \frac{1}{2}\sec x \csc x$

$\frac{1}{2} \cdot \frac{1}{\cos x} \cdot \frac{1}{\sin x}$

$\frac{1}{2\cos x \sin x}$

$\csc 2x = \frac{1}{\sin 2x}$

Assignment: p. 432

12 - 22 evens, 32, 34, 36, 49, 50